A non-geodesic motion in the R^{-1} theory of gravity tuned with observations

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Abstract

In the general picture of high order theories of gravity, recently, the R^{-1} theory has been analyzed in two different frameworks. In this letter a third context is added, considering an explicit coupling between the R^{-1} function of the Ricci scalar and the matter Lagrangian. The result is a non-geodesic motion of test particles which, in principle, could be connected with Dark Matter and Pioneer anomaly problems.

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The accelerated expansion of the Universe, which is today observed, shows that cosmological dynamic is dominated by the so called Dark Energy which gives a large negative pressure. This is the standard picture, in which such new ingredient is considered as a source of the *right hand side* of the field equations. It should be some form of un-clustered non-zero vacuum energy which, together with the clustered Dark Matter, drives the global dynamics. This is the so called "concordance model" (ΛCDM) which gives, in agreement with the Cosmic Microwave Background Radiation (CMBR), Dim Lyman Limit Systems (LLS) and type la supernovae (SNeIa) data, a good trapestry of the today observed Universe, but presents several shortcomings as the well known "coincidence" and "cosmological constant" problems [1]. An alternative approach is changing the *left hand side* of the field equations, seeing if observed cosmic dynamics can be achieved extending general relativity [2, 3, 4, 5, 6]. In this different context, it

is not required to research candidates for Dark Energy and Dark Matter, that, till now, have not been found, but only the "observed" ingredients, which are curvature and baryonic matter, have to be taken into account. Considering this point of view, one can think that gravity is not scale-invariant [7] and a room for alternative theories is present [8, 9, 10, 26, 27, 29]. In principle, the most popular Dark Energy and Dark Matter models can be achieved considering $f(R)$ theories of gravity [7, 11], where R is the Ricci curvature scalar.

In this picture even the sensitive detectors for gravitational waves, like bars and interferometers (i.e. those which are currently in operation and the ones which are in a phase of planning and proposal stages) [12, 13, 31, 33], could, in principle, be important to confirm or rule out the physical consistency of general relativity or of any other theory of gravitation. This is because, in the context of Extended Theories of Gravity, some differences between General Relativity and the others theories can be pointed out starting by the linearized theory of gravity [14, 15, 16, 17, 28, 32].

In the general picture of high order theories of gravity, recently, the R^{-1} theory has been analyzed in two different frameworks [15, 18]. In this letter a third context is added, considering an explicit coupling between the R^{-1} function of the Ricci scalar and the matter Lagrangian. The result is a non-geodesic motion of test particles which, in principle, could be connected with Dark Matter and Pioneer anomaly problems.

In [15] the high order action of the R^{-1} theory of gravity

$$S = \int d^4x\sqrt{-g}R^{-1} + \mathcal{L}_m, \tag{1}$$

has been analyzed in a context of production and potential detection of gravitational waves, while in [18] a cosmological application of such action has been performed.

Equation (1) is a particular choice with respect to the well known canonical one of general relativity (the Einstein - Hilbert action [19, 20]) which is

$$S = \int d^4x\sqrt{-g}R + \mathcal{L}_m. \tag{2}$$

Now let us consider a third action including a coupling between the R^{-1} function of the Ricci scalar and the matter Lagrangian:

$$S = \int d^4x\sqrt{-g}(R^{-1} + R^{-1}\mathcal{L}_m + \mathcal{L}_m). \tag{3}$$

Cleary, this feature implies a breakdown of Einstein's equivalence principle, i.e. a non geodesic motion of test particles [21].

If the gravitational Lagrangian is nonlinear in the curvature invariants, the Einstein field equations are an order higher than second [5, 6, 8, 14, 15]. For this reason such theories are often called higher-order gravitational theories. This is exactly the case of the action (3).

If one varies this action with respect to $g_{\mu\nu}$ (see also refs. [8, 14, 15] for a parallel computation) the field equations are obtained (note that in this paper we work with $G = 1$, $c = 1$ and $\hbar = 1$):

$$
\begin{array}{c}
-R^{-2}R_{\mu\nu} - \frac{R^{-1}}{2}g_{\mu\nu} + \bigtriangledown_\mu \bigtriangledown_\nu R^{-2} - g_{\mu\nu}\Box R^{-2} = \\
= 2R^{-2}\mathcal{L}_m R_{\mu\nu} - 2(\bigtriangledown_\mu \bigtriangledown_\nu - g_{\mu\nu}\Box)R^{-2}\mathcal{L}_m + (1 + R^{-1})T^{(m)}_{\mu\nu},
\end{array} \tag{4}
$$

where $\Box$ is the d' Alembertian operator and $T^{(m)}_{\mu\nu}$ is the ordinary stress-energy tensor of the matter.

Following the analysis in [22], let us compute the covariant derivative of the field equations (4) , together with the Bianchi identity [19]

$$\bigtriangledown^\mu G_{\mu\nu} = 0 \tag{5}$$

and with the identity

$$(\Box \bigtriangledown_\nu - \bigtriangledown_\nu \Box)R^{-2} = R_{\mu\nu} \bigtriangledown^\mu R^{-2}, \tag{6}$$

obtaining

$$\bigtriangledown^\mu T^{(m)}_{\mu\nu} = \frac{-R^{-2}}{R^{-1}+1}(g_{\mu\nu}\mathcal{L}_m - T^{(m)}_{\mu\nu}) \bigtriangledown^\mu R. \tag{7}$$

From the last equation it seems that energy conservation breaks down [21]. This apparent shortcoming can be avoided in the following way. Writing down, explicitly, the Einstein tensor in equation (4) one gets

$$
\begin{array}{c}
G_{\mu\nu} = -R^2 \bigtriangledown_\mu \bigtriangledown_\nu R^{-2} + R^2 g_{\mu\nu}\Box R^{-2} + R g_{\mu\nu} \\
+2\mathcal{L}_m R_{\mu\nu} - 2R^2(\bigtriangledown_\mu \bigtriangledown_\nu - g_{\mu\nu}\Box)R^{-2}\mathcal{L}_m + (R^2 + R)T^{(m)}_{\mu\nu}.
\end{array} \tag{8}
$$

Then, one can introduce a "total" stress-energy tensor

$$
\begin{array}{c}
T^{(tot)}_{\mu\nu} \equiv -\frac{1}{8\pi}\{R^2 \bigtriangledown_\mu \bigtriangledown_\nu R^{-2} + R^2 g_{\mu\nu}\Box R^{-2} + R g_{\mu\nu} \\
+2\mathcal{L}_m R_{\mu\nu} - 2R^2(\bigtriangledown_\mu \bigtriangledown_\nu - g_{\mu\nu}\Box)R^{-2}\mathcal{L}_m + (R^2 + R)T^{(m)}_{\mu\nu}\}.
\end{array} \tag{9}
$$

In this way the field equations assume the well known Einsteinian form

$$G_{\mu\nu} = 8\pi T^{(tot)}_{\mu\nu}. \tag{10}$$

In the "total" stress-energy tensor (9) a *curvature* contribution is added and mixed to the *material* one. This is because the high order terms contribute, like sources, to the field equations and can be considered like *effective fields* (see ref. [25] for details).

Thus, using equation (10), equation (5) gives

$$\bigtriangledown^\mu T^{(tot)}_{\mu\nu}, = 0 \tag{11}$$

i.e. the conservation of the total energy (*material* plus *curvature* plus mixed terms). Moreover, using equation (9), equation (11) gives directly equation (7).

With the goal of testing the motion of test particles in the model, one can introduce the well known stress-energy tensor of a perfect fluid [19, 20, 22]

$$T_{\mu\nu}^{(m)} \equiv (\epsilon + p)u_\mu u_\nu - pg_{\mu\nu}. \tag{12}$$

Because two astrophysical examples will be considered (the first in the galaxy and the second in the Solar System), this simplest version of a stress-energy tensor for the matter, which concerns "dust" (i.e. stars at the galactic scale, planets and spacecrafts at the Solar System scale), can be used in a good approximation [19, 20, 25]. In equation (12) ϵ is the proper energy density, p the pressure and u_μ the fourth-velocity of the particles.

Now, following [22], one defines the *projector operator*

$$P_{\mu\alpha} \equiv g_{\mu\alpha} - u_\mu u_\alpha, \tag{13}$$

the contraction $g^{\alpha\beta}P_{\mu\beta}$ can be applied to equation (7), obtaining

$$\frac{d^2x^\alpha}{ds^2} + \tilde{\Gamma}^\alpha_{\mu\nu}\frac{dx^\mu}{ds}\frac{dx^\nu}{ds} = F^\alpha. \tag{14}$$

The presence of the extra force

$$F^\alpha \equiv (\epsilon + p)^{-1}P^{\alpha\nu}[(\frac{-R^{-2}}{R^{-1}+1})(\mathcal{L}_m + p) \bigtriangledown_\nu R + \bigtriangledown_\nu p] \tag{15}$$

shows that the motion of test particles is non-geodesic. It is also simple to see that

$$F^\alpha \frac{dx_\alpha}{ds} = 0, \tag{16}$$

i.e. the extra force is orthogonal to the four-velocity of test masses.

Taking the Newtonian limit in three dimensions of equation (14) one obtains

$$\overrightarrow{a}_{tot} = \overrightarrow{a}_n + \overrightarrow{a}_{ng} \tag{17}$$

where the total acceleration $\overrightarrow{a}_{tot}$ is given by the ordinary Newtonian acceleration $\overrightarrow{a}_n$ plus the acceleration $\overrightarrow{a}_{ng}$ which is due to the extra force (non-geodesic).

Using equation (17) and a bit of three-dimensional geometry the Newtonian acceleration $\overrightarrow{a}_n$ can be written as

$$\overrightarrow{a}_n = \frac{1}{2}(a_{tot}^2 - a_n^2 - a_{ng}^2)\frac{\overrightarrow{a}_{tot}}{a_{ng}a_{tot}}. \tag{18}$$

In the limit in which $\overrightarrow{a}_{ng}$ dominates (i.e. $a_n \ll a_{tot}$) it is

$$\tag{19}$$

$$a_n \simeq \frac{a_{tot}\overrightarrow{a}_{tot}}{2a_{ng}}(1 - \frac{a_{ng}^2}{a_{tot}^2}). \tag{20}$$

Defining [22, 23, 24, 30]

$$a_e^{-1} \equiv \frac{1}{2a_{ng}}(1 - \frac{a_{ng}^2}{a_{tot}^2}), \tag{21}$$

equation (20) becomes

$$\overrightarrow{a}_n \simeq \frac{a_{tot}}{a_e}\overrightarrow{a}_{tot}. \tag{22}$$

From equation (22) one gets

$$a_{tot} \simeq (a_e a_n)^{\frac{1}{2}}. \tag{23}$$

Because the standard Newtonian acceleration is

$$\overrightarrow{a}_n = \frac{M}{r^2}\widehat{u}_r, \tag{24}$$

the total acceleration results

$$\overrightarrow{a}_{tot} = \frac{(a_e M)^{\frac{1}{2}}}{r}\widehat{u}_r = \frac{v_r^2}{r}\widehat{u}_r, \tag{25}$$

where

$$v_r = (a_e M)^{\frac{1}{4}} \tag{26}$$

is the rotation velocity of a test mass under the influence of the non-geodesic force.

The extra force has environmental nature, thus only phenomenology can help us in its identification.

In a galactic context it is natural to identify a_e with $a_0 \simeq 10^{-10} m/s$, which is the acceleration of Milgrom used in the theoretical context of Modified Newtonian Dynamics to achieve Dark Matter into galaxies [22, 24].

From another point of view, in the Solar System, if the anomaly in Pioneer acceleration is not generated by systematic effects, but a real effect is present [22, 24], one can in principle put

$$a_e = a_{Pi} \simeq 8.5 \times 10^{-10} m/s^2. \tag{27}$$

Thus, the introduced approach allows for a unified explanation of the two effects.

Conclusions

In this letter the R^{-1} theory of gravity has been analyzed in a third context which has been added to two different frameworks recently seen in the general picture of high order theories of gravity. An explicit coupling between the R^{-1} function of the Ricci scalar and the matter Lagrangian has been performed. The result is a non-geodesic motion of test particles which, in principle, could be connected with Dark Matter and Pioneer anomaly problems.

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